Math 3 **1.7 Absolute Value Inequalities** Unit 1

SWBAT write and solve absolute value inequalities and graph solutions on a number line.

Case #1: Less Than <

Suppose you're asked to graph the solution to $|x| < 3$. The solution is going to be all the points that are less than three units away from zero. Look at the number line:

Translating this picture into algebraic symbols, you find that the solution is $(-3, 3)$.

This pattern for "less than" absolute-value inequalities always holds: Given the inequality $|x| < a$, the solution is always of the form $-a < x < a$. Even when the exercises get more complicated, the pattern still holds.

Case #2: Greater Than >

The other case for absolute value inequalities is the "greater than" case. Let's first return to the number line, and consider the inequality $|x| > 2$.

Translating this picture into algebraic symbols, you find that the solution is $(-\infty, -2) \cup (2, \infty)$. That is, the solution is TWO inequalities, not one. **DO NOT** try to write this as one inequality.

The pattern for "greater than" absolute value inequalities always holds: the solution is always in two parts. Given the inequality $|x| > a$, the solution always starts by splitting the inequality into two pieces: $x < -a$ or $x > a$.

Example 1: Solve and graph the solution on a number line: $|2x + 3| < 6$

$2x+3 < 6$ $\quad$ $2x+3 > -6$

$2x < 3$ $\quad$ $2x > -9$

$x < 1.5$ $\quad$ $x > -4.5$

$\boxed{(-4.5, 1.5)}$

All the values of x that make |2x+3| less than 6 units from zero.

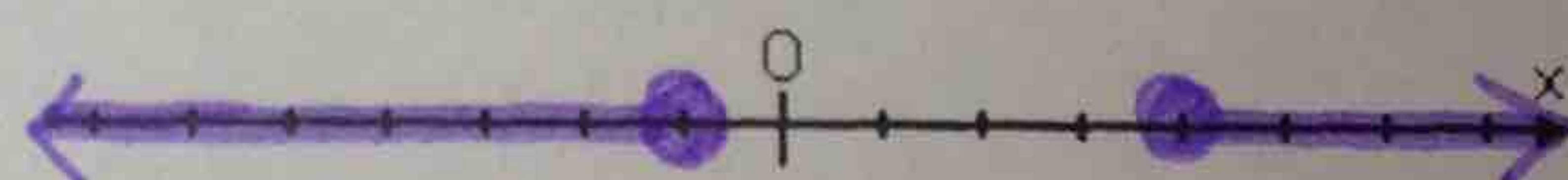
Example 2: Solve and graph the solution on a number line: $|2x - 3| \geq 5$

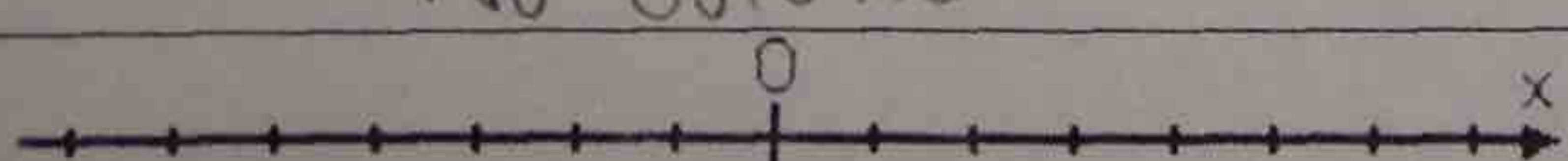
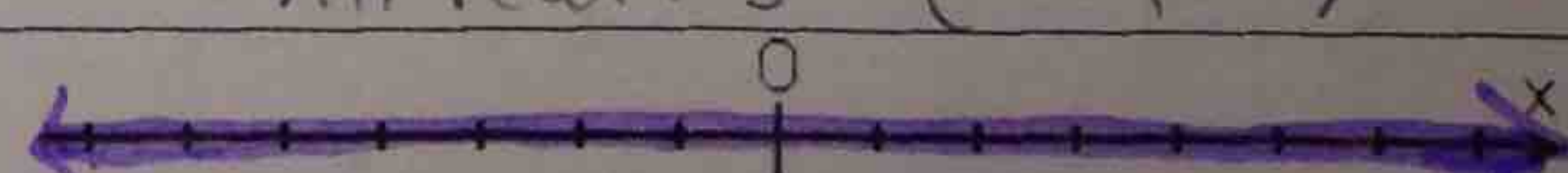
$2x-3 \geq 5$ $\quad$ $2x-3 \leq -5$

$2x \geq 8$ $\quad$ $2x \leq -2$

$x \geq 4$ $\quad$ $x \leq -1$

$\boxed{(-\infty, -1] \cup [4, \infty)}$



Special Case #1: Less than a Negative	**Special Case #2: Greater than a Negative**
Example: $\lvert x + 2\rvert < -1$	Example: $\lvert x - 2\rvert > -3$
Solution: No solution	Solution: All real #s $(-\infty, \infty)$
	

No absolute value will end up negative, nor less than a negative.

All A.V. will be positive, and all positive #s are bigger than any negative

Practice: Solve and graph each solution on a number line.

a) $|-9+p|+5<24$

$|-9+p|<19$

$-9+p<19$ $\quad$ $-9+p>-19$

$p<28$ $\quad$ $p>-10$

$\boxed{(-10, 28)}$

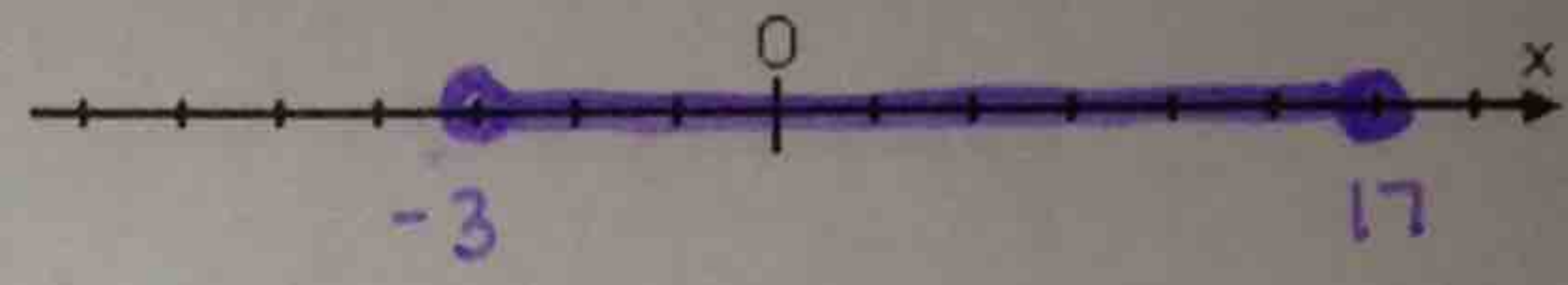
b) $|7-x|+2\le 12$

$|7-x|\le 10$

$7-x\le 10$ $\quad$ $7-x\ge -10$

$-1x\le 3$ $\quad$ $-1x\ge -17$

$x\ge -3$ $\quad$ $x\le 17$



$\boxed{[-3, 17]}$

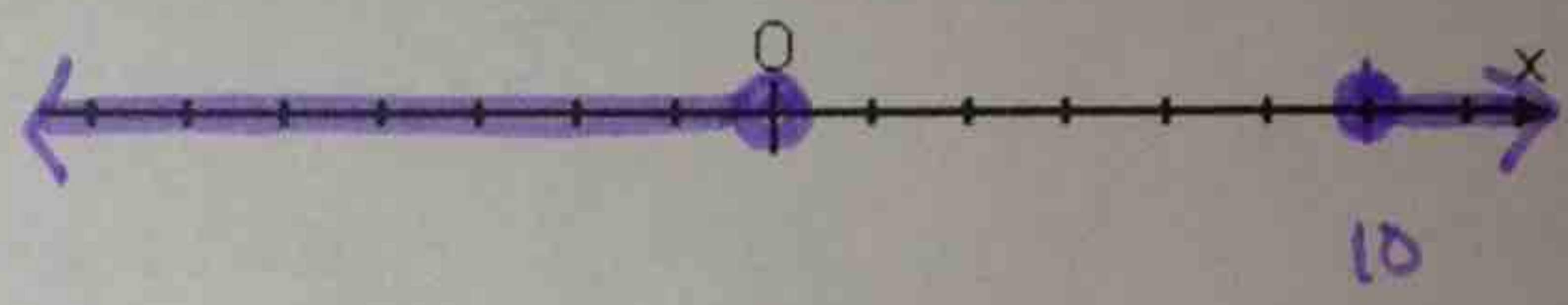
c) $|5-x|+4\ge 9$

$|5-1x|\ge 5$

$5-1x\ge 5$ $\quad$ $5-1x\le -5$

$-1x\ge 0$ $\quad$ $-1x\le -10$

$x\le 0$ $\quad$ $x\ge 10$



$\boxed{(-\infty, 0]\cup[10, \infty)}$

Tolerance Word Problems:	$|$actual – ideal$|$ $\le$ tolerance

Example 3: A carpenter is using a lathe to shape the final leg of a hand-crafted table. In order for the leg to fit, it needs to be 150 mm wide, allowing for a margin of error of 2.5 mm. Write an absolute value inequality that models this relationship, and then find the range of widths that the table leg can be.

$|x-150|\le 2.5$

$x-150\le 2.5$ $\quad$ $x-150\ge -2.5$

$x\le 152.5$ $\quad$ $x\ge 147.5$

$\boxed{147.5\text{ mm}\le x\le 152.5\text{ mm}}$

Example 4: A manufacturer allows a maximum of 18.5 oz of cereal and a minimum of 16.25 oz of cereal per box. Write an absolute value inequality that demonstrates the manufacturer's constraints.

$|x-16|\le 0.6$

$x-16\le 0.6$ $\quad$ $x-16\ge -0.6$

$x\le 16.6$ $\quad$ $x\ge 15.4$

$\boxed{15.4\text{ oz}\le x\le 16.6\text{ oz}}$